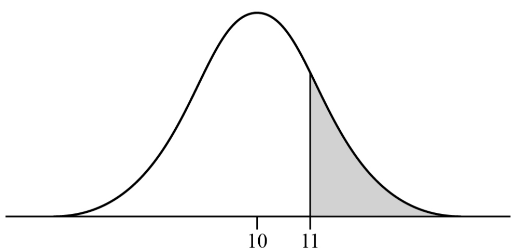
Chapter Review Exercises **(page 416)**

R6.1 (a)
$P(X=5)=1-P(X=1)-P(X=2)-P(X=3)-P(X=4)=1-0.1-0.2-0.3-0.3=0.1.$
(b) Pain score is a discrete random variable because it takes a fixed set of values with gaps in between.
(c) $P(X \le 2) = P(X = 2) + P(X = 1) = 0.2 + 0.1 = 0.3$. $P(X < 2) = P(X = 1) = 0.1$. There are not the same because the outcome $X = 2$ is included in the first calculation but not the second.
(d) $\mu_X = 1(0.1)+2(0.2)+3(0.3)+4(0.3)+5(0.1)=3.1.$
$\sigma_X^2 = (1-3.1)^2(0.1)+...+(5-3.1)^2(0.1)=1.29,$ so $\sigma_X = \sqrt{1.29} = 1.136.$

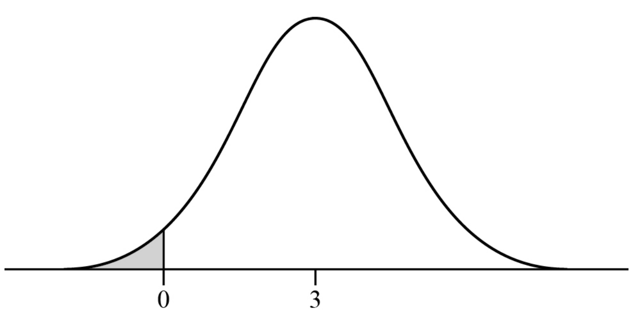
R6.2 (a) Temperature is a continuous random variable because it takes all values in an interval of numbers—there are no gaps between possible temperatures.
(b) $P(X < 540) = P(X \le 540)$ because X is a continuous random variable. In this case, $P(X = 540) = 0$ because the line segment above $X = 540$ has no area.
(c) The mean number of degrees off target is 550 − 550 = 0°C, and the standard deviation stays the same, 5.7°C, because subtracting a constant does not change the variability.
(d) In degrees Fahrenheit, the mean is $\mu_Y = \frac{9}{5}\mu_X + 32 = 1022\,°\text{F}$ and the standard deviation is
$\sigma_Y = \left(\frac{9}{5}\right)\sigma_x = 10.26°\text{F}.$

R6.3 (a) If you were to play many games of 4-Spot Keno, you would get a payout of about \$0.70 per game, on average. If you were to play many games of 4-Spot Keno, the payout amounts would typically vary by about \$6.58 from the mean (\$0.70).
(b) Let Y be the amount of Jerry's payout. Then $Y = 5X$. Therefore
$\mu_Y = 5\mu_X = 5(0.70) = \$3.50$ and $\sigma_Y = 5\sigma_X = 5(6.58) = \$32.90.$
(c) Let W be the amount of Marla's payout. Because she plays 5 separate \$1 games,
$W = X_1 + X_2 + X_3 + X_4 + X_5$ where all of the X's have the same distribution and are independent of each other. This means that $\mu_W = \mu_{X_1} + \mu_{X_2} + \mu_{X_3} + \mu_{X_4} + \mu_{X_5} = 5\mu_X = 5(0.70) = \$3.50$ and
$\sigma_W^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 + \sigma_{X_3}^2 + \sigma_{X_4}^2 + \sigma_{X_5}^2 = 5\sigma_X^2 = 5(6.58)^2 = 216.482,$ so $\sigma_W = \sqrt{216.482} = \$14.71.$
(d) Even though their expected values are the same, the casino would probably prefer Marla since there is less variability in her strategy. They are less likely to get great amounts from her, but also less likely to have to pay great amounts to her.

R6.4 (a) **Step 1: State the distribution and values of interest.** C follows a $N(10, 1.2)$ distribution and we want to find $P(C > 11)$ as shown below. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{11-10}{1.2} = 0.83$. The desired probability is $P(Z > 0.83) = 1 - P(Z \leq 0.83) = 1 - 0.7967 = 0.2033$. *Using technology:* The command normalcdf(lower: 11, upper: 1000, μ: 10, σ: 1.2) gives an area of 0.2023. **Step 3: Answer the question.** There is a 0.2023 probability that a randomly selected cap has a strength greater than 11 inch-pounds.



(b) It is reasonable to assume the cap strength and torque are independent because the machine that makes the caps and the machine that applies the torque are not the same.
(c) T is $N(7, 0.9)$ and C is $N(10, 1.2)$, so $C - T$ is Normal with mean 10 – 7 = 3 inch-pounds and standard deviation $\sqrt{0.9^2 + 1.2^2} = 1.5$ inch-pounds.
(d) **Step 1: State the distribution and values of interest.** $C - T$ follows a $N(3, 1.5)$ distribution and we want to find $P(C < T) = P(C - T < 0)$ as shown below. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{0-3}{1.5} = -2$. The desired probability is $P(Z < -2) = 0.0228$. *Using technology:* The command normalcdf(lower: –1000, upper: 0, μ: 3, σ: 1.5) gives an area of 0.0228. **Step 3: Answer the question.** There is a 0.0228 probability that a randomly selected cap will break when being fastened by the machine.



R6.5 (a) Check the BINS: Binary? "Success" = candy is orange and "Failure" = candy is not orange. Independent? The sample of size $n = 8$ is less than 10% of the large bag, so we can assume the outcomes of trials are independent. Number? We are choosing a fixed sample of $n = 8$ candies. Success? The probability of success remains constant at $p = 0.20$. This is a binomial setting, so X has a binomial distribution with $n = 8$ and $p = 0.20$.
(b) $\mu_X = np = 8(0.2) = 1.6$. If we were to select many samples of size 8, we would expect to get about 1.6 orange M&M'S, on average.

(c) $\sigma_X = \sqrt{np(1-p)} = \sqrt{8(0.2)(0.8)} = 1.13$. If we were to select many samples of size 8, the number of orange M&M'S would typically vary by about 1.13 from the mean (1.6).

R6.6 (a) $P(X=0) = \binom{8}{0}(0.2)^0(0.8)^8 = 0.1678$. Because the probability is not that small, it would not be surprising to get no orange M&M'S in a sample of size 8.

(b) $P(X \geq 5) = \binom{8}{5}(0.20)^5(0.80)^3 + \text{L} + \binom{8}{8}(0.20)^8(0.80)^0 = 0.0104$. Using technology: 1 – binomcdf(trials: 8, *p*: 0.20, *x* value: 4) = 1 – 0.9896 = 0.0104. Because the probability is small, it would be surprising to find 5 or more orange M&M'S in a sample of size 8.

R6.7 Let *Y* be the number of spins to get a "wasabi bomb." *Y* is a geometric random variable with $p = \frac{3}{12} = 0.25$. $P(Y \leq 3) = (0.75)^2(0.25) + (0.75)(0.25) + 0.25 = 0.5781$. *Using technology*: geometcdf(*p*: 0.25, *x* value: 3) = 0.5781. There is a 0.5781 probability that it takes 3 or fewer spins to get a wasabi bomb.

R6.8 (a) Let *X* be the number of heads in 10,000 tosses. Assuming the coin is balanced, *X* has a binomial distribution with $n = 10{,}000$ and $p = 0.5$. Therefore $\mu_X = np = 10{,}000(0.5) = 5{,}000$ and $\sigma_x = \sqrt{np(1-p)} = \sqrt{10{,}000(0.5)(0.5)} = 50$.

(b) The conditions are met to approximate the binomial distribution with a Normal distribution because both $np = 10{,}000(0.5) = 5{,}000$ and $n(1-p) = 10{,}000(0.5) = 5000$ are at least 10.

(c) **Step 1: State the distribution and values of interest.** *X* follows an approximately Normal distribution with mean 5000 and standard deviation 50. We want to find $P(X \leq 4933$ or $X \geq 5067)$ as shown below. **Step 2: Perform calculations. Show your work.** The standardized scores for the boundary values are $z = \frac{4933-5000}{50} = -1.34$ and $z = \frac{5067-5000}{50} = 1.34$. The desired probability is $P(Z \leq -1.34) + P(Z \geq 1.34) = 0.0901 + 0.0901 = 0.1802$. *Using technology:* The command 1 – normalcdf(lower: 4933, upper: 5067, μ: 5000, σ: 50) gives an area of 0.1802. **Step 3: Answer the question.** Because this probability isn't small, we don't have convincing evidence that Kerrich's coin was not balanced—a difference this far from 5000 could be due to chance alone